useful to the extent that they are symmetries of the theory we are interested in. For example, we will define P so that it is a good symmetry of QED, but there is no way to define it so that it is preserved under the weak interactions. In any representation, we should have $P^2 = T^2 = 1$.

11.5.1 Scalars and vectors

For real scalars, parity should be a symmetry of the kinetic terms $\mathcal{L} = -\frac{1}{2}\phi\Box\phi - \frac{1}{2}m^2\phi^2$ or we are dead in the water. Thus, $P^2 = 1$ (we do not need to use $P^2 = 1$ in the Lorentz group for this argument) and there are two choices:

$$P: \quad \phi(t, \vec{x}) \to \pm\phi(t, -\vec{x}). \tag{11.59}$$

The sign is known as the **intrinsic parity** of a particle. In nature, there are particles with even parity (scalars, such as the Higgs boson) and particles with odd parity (pseudoscalars, such as the π^0). Since the action integrates over all $\vec{x}$, we can change $\vec{x} \to -\vec{x}$ and the action will be invariant.

For complex scalars, the free theory has Lagrangian $\mathcal{L} = -\phi^\star\Box\phi - m^2\phi^\star\phi$, so the most general possibility is

$$P: \quad \phi(t, \vec{x}) \to \eta\,\phi(t, -\vec{x}), \tag{11.60}$$

where η is a pure phase. Note that $P^2 : \phi(t, \vec{x}) \to \eta^2\phi(t, \vec{x})$, so P^2 is an internal symmetry (not acting on spacetime). In principle, the parity phase η can be arbitrary for each particle.

Consider scalar QED with a bunch of particles of different charges. It has a continuous global symmetry under which $\phi \to e^{i\alpha Q}\phi$ for any $\alpha \in \mathbb{R}$ with Q the charge of ϕ. If P^2 is a subgroup of this global charge symmetry, then $\eta^2 = e^{i\beta Q}$. Thus the discrete symmetry $(P')^2 = P^2 e^{-i\beta Q} = \mathbb{1}$ and $P' = Pe^{-i\frac{\beta}{2}Q} : \phi(t, \vec{x}) \to \pm\phi(t, -\vec{x})$ for all particles. Thus, we might as well take the convention that P' is called parity rather than P, and therefore all particles in this theory have parity ± 1.

In the Standard Model, there are three continuous global symmetries: lepton number L (leptons have $L = 1$, everything else has $L = 0$), baryon number B (quarks have $B = \frac{1}{3}$, everything else has $B = 0$) and electric charge Q. If P^2 is a subgroup of the product of these, then $P^2 = e^{i(\alpha B + \beta L + \gamma Q)}$ for some α, β, and γ. We then define $P' = Pe^{-\frac{i}{2}(\alpha B + \beta L + \gamma Q)}$ so that all particles have parity phases of ± 1. Moreover, if the proton has parity -1, we can redefine again by $P'' = (-1)^B P'$ so that proton has parity $+1$. Similarly using up L and Q, we get to pick three parity phases in total, which conventionally are that the proton, neutron and electron all have parity $+1$. See [Weinberg, 1995, p.125] for more details of this argument.

From nuclear physics measurements, it was deduced that the pion, π^0, and its charged siblings, π^+ and π^-, all have parity -1. Then it was very strange to find that a particle called the kaon, K^+, decayed to both two pions and three pions. People thought for a while that the kaon was two particles, the θ^+ (with parity $+1$, which decayed to two pions) and the τ^+ (with parity -1, which decayed to three pions). Lee and Yang finally figured out, in 1956, that these were the same particle, and that parity was not conserved in kaon decays.

For vector fields, P acts as it does on 4-vectors. However, for the free vector theory to be invariant, we only require that

$$P: \quad V_0(t, \vec{x}) \to \pm V_0(t, -\vec{x}), \qquad V_i(t, \vec{x}) \to \mp V_i(t, -\vec{x}). \tag{11.61}$$